Tensions Between Teacher & Student Attention

John Mason[1]

Abstract

The claim made is that there is a fundamental pedagogical tension between the natures of teacher attention and learner attention when the two are interacting. For example, differences between these can be used to account for a great deal of the mis-communication that takes place in classrooms.

Method

My approach is fundamentally phenomenological. I am interested in lived experience, and I find that the best way to communicate mathematical, didactical (topic specific) and pedagogical insights is through evocation via proposing a task-exercise for readers which I have found fruitful, followed by reflecting on and connecting that experience to other experiences and to frameworks of distinctions to be found in the literature.

Outline

After some initial tasks I offer brief remarks about the nature and form of attention as it applies to learning mathematics, followed by further tasks in which it may be possible to relate your experience with my descriptions.

Tasks

I begin with some task-exercises. They offer an opportunity to try to catch yourself shifting attention, whether between different foci, or in the form or nature of how you are attending.

Number Line Scaling

Task 1: Scaling from zero.

Imagine a number-line with the integers marked on it. Imaging a flexible transparent copy lying on top of it.

Imagine scaling (stretching) the number line (the flexible one) by a factor of say 3, keeping 0 fixed. So 0 on the fixed number-line is the centre of the scaling.

Where does 4 on the flexible line end up aligned with on the fixed line? What happens to –2?

Generalise.

Comment

You may have considered drawing a number line in order to help fix those parts that aren't changing. That way you can attend to the parts that are changing (the scaling).

You may have found yourself gazing at the words of the task, however briefly, before discerning details and carrying them out. The same may happen if you use a picture of a number line: you may have gazed at it, briefly, before discerning relevant detail, and then focusing in on that as a new object at which to first gaze and then discern details.

Task 1a: a diversion

Go back to your number-line with the transparent copy on top. Rotate the flexible line through 180° about the point 0.

Develop a succinct way of predicting where any chosen number on the flexible number line ends up on the fixed line.

---

[1] John Mason, Emeritus Professor, The Open University, UK. john.mason @ open.ac.uk.

Mason, J. (2018). Tensions between teacher & student attention. WikiLetter. 9th April 2018.

Comment

Rotating through 180° about 0 has the same effect as multiplying the number by –1. Notice that rotating again through 180° about 0 returns the number to its original position. The action 'multiply by –1', when repeated twice, has no overall effect. In other words $(-1)(-1) = 1$.

There is a whole sequence of useful tasks that can be developed from here, rotating about different points, compounding rotations, and so on, in parallel with the following tasks.

Task 2: Scaling from other points

> Imagine your two number-lines again. This time, stretch the flexible line by a factor of 3 but use 2 as the fixed point, the centre of the scaling.
>
> Where does 4 end up aligned with on the fixed number-line? –1?
>
> Generalise

Comment

There are at least two ways to carry out this action. You can notice that it is the distance from 2 that is multiplied by 3, so 4 goes to (4-2) x 3 + 2 because you calculate the distance to 2, scale it, and then mark that distance off from 2 in the same direction. Alternatively, you can use what you already know: namely how to scale from 0. So translate the number line so that 2 ends up at 0 (you do this by subtracting 2). Now scale by a factor of 3 from 0 (by multiplying by 3). Then translate back again (by adding 2 so that 0 ends up at 2) actual. So 4 goes to (4 – 2) x 3 + 2.

Notice how *not* doing the calculations reveals the structure of the reasoning, where doing the arithmetic would obscure it. Notice also that this act (called *tracking arithmetic*: Mason 2016) makes it easy to apply the same action to any number not simply to 4, by treating 4 as a place holder rather than as a specific number. Replacing 4 by k leads to $(k - 2) \times 3 + 2$ as the place that k ends up. Furthermore, treating 2 as a place holder reveals the effect when scaling from any point as the centre of scaling. Denote the centre of scaling by c. While we are at it, denote the scale factor by σ. Then scaling by a factor of σ from a centre c sends k on the original number-line to position $(k - c)\sigma + c$ on the original number-line

Perceiving and expressing the relationship between a point k on the number-line and its position after a scaling opens up further questions. Suppose scaling has taken place by a known amount but from an unknown centre. If you know where just one point on the number-line ends up, you can work out the scale factor. Suppose you know where two different points on the number-line end up. Can you always work out both the scale factor and the centre?

Interlude on Attention

Each task invites work on two specific numbers (4 and –1) and then on an example of your own. The intention is to provide some specific experience with particular numbers, but not to be content with simply answering the question. Rather the implicit invitation is to *recognise relationships* between the starting number and where it goes, in two or more instances. This can then lead to *perceiving a property* being instantiated, namely, subtract the centre of scaling from your number, multiply by the scale factor, and then add your centre back again. What you are doing is translating, scaling, then translating back again: a common mathematical activity known as *conjugation* or more generally as *bypasses* (Melzak 1983).

Attention functions on different levels (Mason 2002). Here, I have tried to draw your attention (*sic*) to distinctions between the nature or form of attention at the micro level:

> *Holding wholes* (gazing)
>
> *Discerning details*
>
> *Recognising relationships*
>
> *Perceiving properties as being instantiated*
>
> *Reasoning on the basis of agreed properties*

You cannot recognise relationships until you have discerned details that you might the relate in some way. Different details can constitute wholes to be 'held' or gazed at, and details within these may then be discerned. Here lay tensions between teacher and student attention.

Teacher-Learner Tensions

When a teacher carries out a 'worked example' in front of learners, the teacher is aware of the working as a specific instance, an instantiation of something more general. That was certainly the case for me setting out the task, and this enabled me to remember to choose distinct numbers and to use *tracking arithmetic* so as to draw your attention not to the specific answers for the points 4 and –1, but to the relationship between a number and where it neds up under some geometrical action, expressed in algebraic terms as a property being instantiated.

However, a learner is likely to be caught up with the mechanics of the working, with an eye on 'the answer', and so be attending differently, if not also attending to different details. As long as this difference happens, communication is at best fragile: learners may not properly 'hear' what is said, or may overlook what is written.

The claim being made is that if the teacher and learner(s) are attending to different things then there is unlikely to be effective communication. Even if they are both attending to the same thing (such as where 4 goes under a scaling by 3 from 2 as a centre), if they are attending differently, then communication is still likely to be ineffective. By differently, I mean that the teacher may be recognising relationships while the student is trying to discern the details the teacher is referring to, or may even be still gazing at the diagram; the teacher may be perceiving properties as being instantiated, while the student is either still trying to recognise relationships in the particular situation, or may still be discerning details or even gazing at some detail.

It behoves the teacher to watch and listen to learners, to try to detect both what they are attending to, and how, so that as to be able to speak, point, or gesture appropriately. Rushing on is less effective than pausing to provide time for learners to gaze, to discern details and to recognise relationships between discerned objects which themselves may have required some time gazing.

More Tasks

In the light of the elaboration of attention in the previous section, here is an opportunity to try to trap movements of your attention, while encountering some slightly more challenging mathematics.

It seems self-evident that compounding scalings, that is carrying out one after another, has the effect of scaling by the product of the scale factors. If you scale a map, or a number line by scale factor σ, and then scale that by a scale factor ρ, the same overall effect could be achieved by a single scaling by a factor of $\rho\sigma$.

Task 3: Compound Scaling

> Imagine a number-line again, with a flexible line on top of it.
>
> First, scale (by stretching) the number line by a scale factor of, say, 3 keeping the point 2 fixed.
>
> Now scale that by a factor of say 1/7 but keeping fixed the point which is now aligned with the original point 5.
>
> Where does 4 end up? –1?
>
> What single scaling (about what point) will achieve the same effect as compounding these two scalings?

Comment

Intuitively, the overall scale factor must be 3/7. Finding the centre of scaling, the fixed point, is not so easy!

Warning: spoiler alert

Algebraically, let *c* be the location of the overall centre. Scaling about *c* by a factor of 3/7 is achieved for the point *n* through (*n* – *c*)x 3/7 + *c*. This can conveniently be written as *n* x 3/7 + (1 – 3/7) *c*. But it is arrived at by first enacting (*n* – 2) x 3 + 2, and then performing the second scaling on this, which sends it to ((*n* – 2) x 3 + 2) – 5)x 1/7 + 5. This can be re-written as *n* x 3/7 + 2(1 – 3) x 1/7 + 5(1 – 1/7). Since these two expressions have to be the same whichever value of *n* is used, it must be that (1 – 3/7)*c* = 2(1 – 3)/7 + 5(1 – 1/7). From this *c* can be found.

Notice that the particular numbers have been chosen to be all different, and arithmetic calculations have been avoided, so that each of the numbers can be treated as a place holder in the process of *tracking arithmetic* as performed earlier. So if the first centre is at c_1 with scale factor σ_1, and the second is at c_2 with scale factor σ_2, then the final formula can be interpreted more generally as

$$(1 - \sigma_2\sigma_1)c = c_1(1 - \sigma_1)\sigma_2 + c_2(1 - \sigma_1) \text{ or}$$

$$c = \frac{(1 - s_1)(c_1 s_2 + c_2)}{1 - s_1 s_2}$$

It is evident from the lack of symmetry in the roles of c_1 and c_2 that there will be a different answer if c_1 and c_2 are interchanged along with σ_1 and σ_2. In other words, compounding scalings has the scale factors commuting, but not the centres, when they are different.

What happens if $\sigma_1\sigma_2 = 1$? Where would the centre for the combined scaling be?

Comment

It was probably very tempting to keep on reading rather than to try to express the conditions which determine the location of the single centre which achieves the same result as the two scalings from different centres. If so, that is an opportunity lost, because your sense of how it works is at best second hand, perhaps even third hand given that I have processed it pedagogically in setting it down here. Plunging on to what follows before checking with your own experience would throw away an important opportunity, which is but an instance of a general pedagogical observation that *self-explanation* or *personal narrative* (Renkl 2002; Hodds, Alcock & Inglis 2015) makes an important contribution to enriching the web of connections which is what people seem to mean by 'learning'.

Task 3a: personal narrative

> Here is an opportunity to check your comprehension and appreciation: where will the overall centre be for scaling in the other order, and under what conditions can those combined centres be the same?

The most important pedagogic action here, perhaps apart from articulating aspects of attention, is the use of tasks to stimulate learners to form their own narrative, their own story about what is going on, and to explore around situation by changing conditions. You can only say that you begin to appreciate and comprehend (understand) something when you can place it in a more general context.

References

Hodds, A. Alcock, L. & Inglis, M. (2014). Self-Explanation Training Improves Proof Comprehension. *Journal for Research in Mathematics Education*. 45(1) p62-101.

Mason, J. (2002). *Researching Your own Practice: The Discipline of Noticing*. RoutledgeFalmer, London.

Mason, J. (2016). Overcoming the Algebra Barrier: being particular about the general, and generally looking beyond the particular, in homage to Mary Boole. In S. Stewart (Ed.) *And the Rest is Just Algebra*. Springer p97-117.

Melzak, Z. (1983). *Bypasses: a simple approach to complexity*. New York: Wiley.

Renkl, A. (2002) Worked-out examples: Instructional explanations support learning by self-explanations. *Learning and Instruction, 12*, 529–556.