Empowering Children to Think Algebraically

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Many people have had an encounter with algebra at some point in school mathematics that has left them with a sense of failure and even dread. Schoenfeld's (1995) observation over two decades ago that algebra served as a "gatekeeper" pointed to a larger reality, at least in schools in the United States[2], that algebra as traditionally taught had pushed too many students out of career opportunities in STEM-related fields. This was particularly felt among students in underrepresented groups (Moses & Cobb, 2001). The resulting motivation to transform algebra from an "engine of inequity" to one of mathematical power (Kaput, 1998) created a watershed moment in which teaching and learning algebra was reconceptualized as a longitudinal approach that would begin in the elementary grades and leverage young children's informal ideas about mathematical structure and relationships more organically into formal algebraic thinking.

The prospect of this transformation raised important questions that have been the subject of much research: What would such an approach to teaching and learning algebra look like in the elementary grades?, Would young children be able to think in ways that have traditionally been viewed as possible only for older students?, What impact would such an approach have, if any, on students' algebra-readiness for secondary grades?, and Given that elementary teachers would be central to reform in algebra education, how should they be prepared to build authentic algebra learning environments in ways that would not reinforce past student failures?

A premise for answering these questions requires characterizing the 'algebra' that we want young children to learn. As Kaput (2008) argued, "What we think algebra *is* has a huge bearing on how we approach it—as teachers, administrators, researchers, teacher-educators, curriculum developers, framework writers, instructional material evaluators, assessment writers, policy makers, and so forth." (p. 8) To this end, we use Kaput's (*ibid*) content analysis of algebra as a conceptual framework in our work, organizing (early) algebra[3] around four core algebraic thinking practices: *generalizing*, *representing*, *justifying* and *reasoning with* mathematical structure and relationships (Blanton, Brizuela et al., 2018).

Over the past decade, we have worked to identify trajectories in children's algebraic thinking around these core practices and, by synthesizing this work with other existing research, design interventions for elementary grades that might mitigate algebra's gatekeeper effect in school mathematics. What we have found about the strength of young children's algebraic thinking has been both impressive and—to some extent—surprising. While thinking algebraically is not without its challenges for students of any age, we are encouraged that engaging children in these core practices from kindergarten—at the start of formal schooling—and beyond is both possible and promising. In what follows, I highlight some of our broader findings and then focus on two tasks that can both serve as straightforward entry points for instruction and reveal important insights into children's algebraic thinking.

Developing Effective Tools to Challenge Algebra's "Gatekeeper" Effect

A major goal of our work has been to design interventions from which we could measure the impact of sustained, longitudinal early algebra instruction on children's understanding of core algebraic concepts and practices and their readiness for algebra in middle grades. Our resulting *LEAP Curriculum* is a sequence of 18 lessons for each of grades 3–5 that engages students in the four core algebraic thinking practices across different important areas of mathematical content using increasingly sophisticated ideas, content, and representations. Lessons emphasize the development of meaning through students' written and oral explanations of their mathematical thinking.

In experimental studies, we have found that students who are taught LEAP as part of their regular classroom instruction significantly outperform their peers who are taught a more traditional, arithmetic-focused curriculum (Blanton, Isler et al., 2017; Blanton, Stroud et al., 2018) in their understanding of core algebraic concepts and

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[2] Our research has occurred only within schools in the United States, so I am writing here from the perspective of teaching and learning algebra within that context.

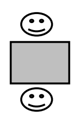
[3] By early algebra, I mean algebraic thinking in elementary or primary grades.

practices. LEAP students are significantly more able to interpret the equal sign as a relational symbol, recognize properties of operations and represent them with variable notation, build arguments for mathematical claims that are increasingly general and more sophisticated than arguments that only test numerical examples, recognize unknown quantities in mathematical situations and represent them in algebraic expressions, and generalize functional relationships and represent them with words and variable notation. Surprisingly, we have also found that students are more successful in representing functions with variable notation than with their own words, underscoring the sometimes-contested argument that variable notation can be an important tool in young children's algebraic reasoning. We have found that students in lower elementary grades can transition from an initial primitive "pre-variable/pre-symbolic" view of variable and variable notation in which they neither perceive a variable quantity in a problem situation nor understand how to represent it with variable notation, to a view of variable as a varying quantity that can be acted on as a mathematical object (Sfard, 1991) in representing relationships between quantities (Blanton, Brizuela, Gardiner, & Sawrey, 2017).

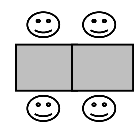
Consider the following example of growth in students' ability to *generalize* and *represent* relationships—two core algebraic thinking practices. In the "Brady Problem", students were asked a variety of questions, including whether they could find a relationship between the number of desks and the number of students that could be seated at the desks and represent this relationship with words and variable notation.

Brady is celebrating his birthday at school. He wants to make sure he has a seat for everyone. The desks where students are seated are square-shaped.

He can seat 2 people at one desk in the following way:



If he joins another desk to the first one, he can seat 4 people:



If he joins another desk to the second one, he can seat 6 people:

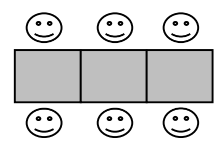


Table records one student's responses across grades 3–5.

Timepoint	Relationship (Words)	Relationship (Variables)
Beginning of Grade 3	I Put 3 desks and four kids on each desk and there is 12 kid	?
End of Grade 3	the # of desks count by 1 # of people count by 2.	?
End of Grade 4	It is alway plus its self.	A x 2
End of Grade 5	The number of people is double the number of desks.	d x 2 = p

The table shows increasingly sophisticated ideas that mature from not recognizing a pattern or relationship at all at the beginning of third grade, to a clear understanding and representation (in words and variables) of a functional correspondence relationship in fifth grade.

Our most compelling finding, however, is that the effectiveness of the LEAP curriculum also holds for those students within at-risk populations: LEAP students from economically disadvantaged and demographically diverse schools significantly outperform their peers in similar schools where only the regular arithmetic curriculum is taught (Blanton, Stroud et al., 2018). For us, this shows promise in LEAP's potential (or that of *any* early algebra curriculum) to ameliorate algebra's gatekeeper effect, leading to more opportunities for success in mathematics, for *all* students.

Two Simple Tasks that Reveal Important Insights into Children's Algebraic Thinking

Elementary teachers are often not sufficiently prepared to integrate early algebra into their daily practice in routine ways .Let's consider two tasks that might be productive entry points for teachers.

Equal Sign Tasks.

A *relational* understanding of the equal sign as a symbol that denotes that two mathematical objects are equivalent is foundational to both arithmetic and algebra. However, it is well documented that students in elementary grades—and beyond—often view this symbol *operationally*, as a prompt to perform the computation indicated in the expression to the left of the equal sign (e.g., Stephens, Ellis, Blanton, & Brizuela, 2017).

We (and others) use open equations as a way to ascertain whether students hold an operational or relational view of the equal sign. Consider a simple task such as 7 + 3 = ____ + 4, designed so that there are operations on both sides of the equal sign and a missing value placed to the right of the equal sign. Students who interpret the symbol '=' operationally will claim that the missing value is 10 (that is, 7 + 3) or 14 (that is, 7 + 3 + 4). Students who interpret the symbol relationally will find the missing value to be 6 by one of two ways: Adding 7 and 3 and subtracting 4 from the total, or recognizing that 4 is 1 more than 3, so the missing value must be 1 less than 7.

The beauty of such tasks is that not only do they reveal significant aspects of students' algebraic thinking (or lack of it), they are also simple to build into instruction. Moreover, we have found that they serve as a productive entry point for teacher professional development: Asking teachers to gather data about their students' responses to such tasks can be a powerful starting point for discussions about what it means for children to think algebraically. And questions such as How did students respond to this task? What does it say about their understanding of equivalence? How could such tasks be integrated thoughtfully into instruction? What tools or manipulatives might support the development of a relational view of this symbol? and Why is students' understanding of equivalence important in arithmetic and algebra? can help teachers probe more deeply what it means to teach algebraic concepts to young children. Because teachers are often surprised that students struggle so significantly with such a "simple" task, we find that these tasks can provide an easy "hook" to engage teachers in thinking about algebra in the elementary grades.

Mathematizing an unknown. Children in elementary grades are quite familiar with tasks in which they asked are to operate on *knowns* to find an *unknown*. Tasks such as "If Marta has 4 pieces of candy and her mother gives her 6 more pieces, how many pieces of candy does she have all together?" are prominent in elementary grades mathematics and can certainly be used in productive ways to build students' arithmetic thinking. But what if one of the *knowns* in a task such as this is made unknown: "If Marta has a box of candies and her mother gives her 6 more pieces, how would you describe the number of pieces she has all together?" (This type of task is adapted from Carraher, Schliemann, & Schwartz;2008). Although the transformation of this task is simple (making the known number of candies in Marta's box unknown), the opportunity for young children to mathematize a problem where an unknown exists *as part of the formulation of the problem* is critical. The abundance of literature that points to the difficulty students have with variable (and variable notation) in formal algebra in later grades underscores the need for children to begin to engage with mathematical situations, such as the revised task given here, in which they are not operating entirely on *knowns.*

In the revised problem about Marta's candy, young children will usually initially assign a value to the number of candies in Marta's box (e.g., Marta has 5 pieces of candy in the box) even though it violates the problem situation. With instruction, students begin to see that the number of candies can be one of a (finite) range of values, then finally recognize that the quantity varies and can, in theory, be any number (although given the physical context, we expect it to be a reasonable number in practice). Instruction that supports students in exploring this ambiguity – where the number of a quantity is not a fixed, known amount – is important in building their comfort with this central algebraic idea of unknowns.

The two task types given here can be a simple starting point in engaging teachers in the work of teaching and learning algebra in elementary grades. We have found that this process is an inspiring journey in which early algebra comes to be viewed by teachers as innovative and inclusive, transforming how they view students, teaching, and mathematics. Perhaps more importantly, it also empowers students, transforming how they view themselves as mathematical thinkers.

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